Relationship between Correlation and Volatility

in Closely-Related Assets

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The purpose of this mini research paper is to address in a more quantitative fashion the relationship between the correlation of two highly correlated assets such as the S&P 500 index and the FTSE-100 index, and their individual volatilities.

It is intuitively clear (and traders would confirm this based on their market experience) that assets, such as the S&P 500 index and FTSE-100 index are strongly related to each other. For example, their corresponding futures markets (ES and Z_, respectively) overlap in trading: ES trades from 6:00PM through 4:15PM, and Z_ trades from 3AM to 4PM EST. This provides a good 13 hours of overlap time, when these two markets co-exist. It is also known that they are often used as proxies for equity market risk in their corresponding liquid times, which are shifted with respect to each other: Z_ has its peak liquidity from around 3AM going down to 11:30AM, whereas ES has two peak liquidity times: one around 9:30AM to 10:30AM, another from 2PM to 4:15PM.

We chose these two markets simply as an example, though very similar arguments and experimental results hold when one replaces FTSE-100 with other assets such as DJ EURO STOXX, Dax, CAC-40, Swiss SMI, etc.

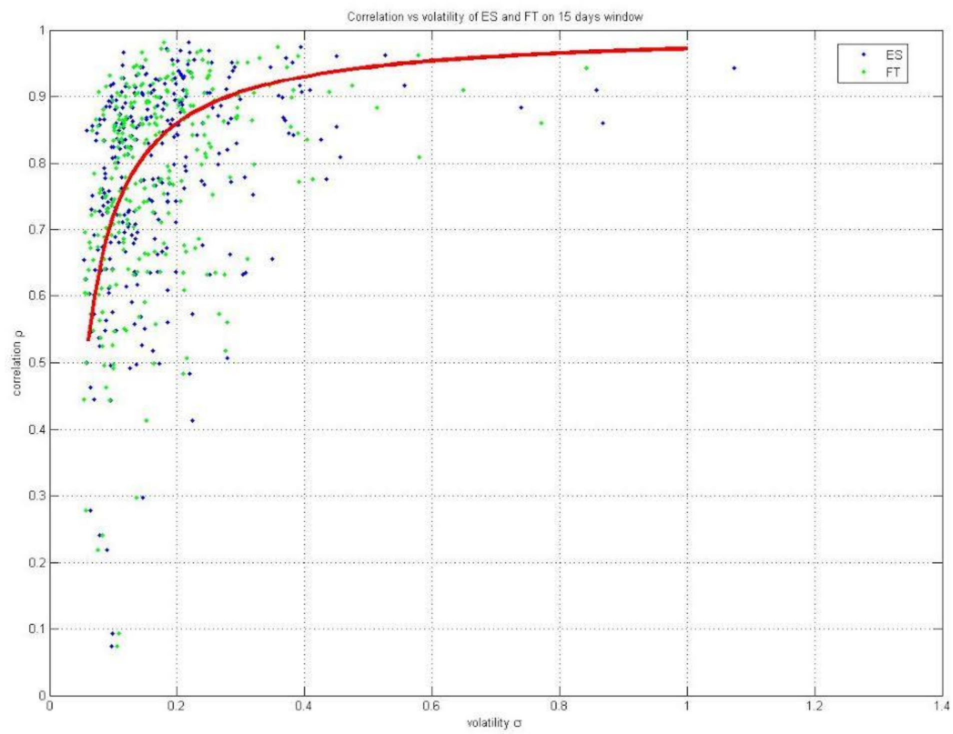
Indeed, on average over very long time intervals (say, over 20 years) on daily sampling, FTSE-100 and S&P 500 are about 65% correlated to each other. Everywhere in this paper correlation refers to the correlation coefficient.

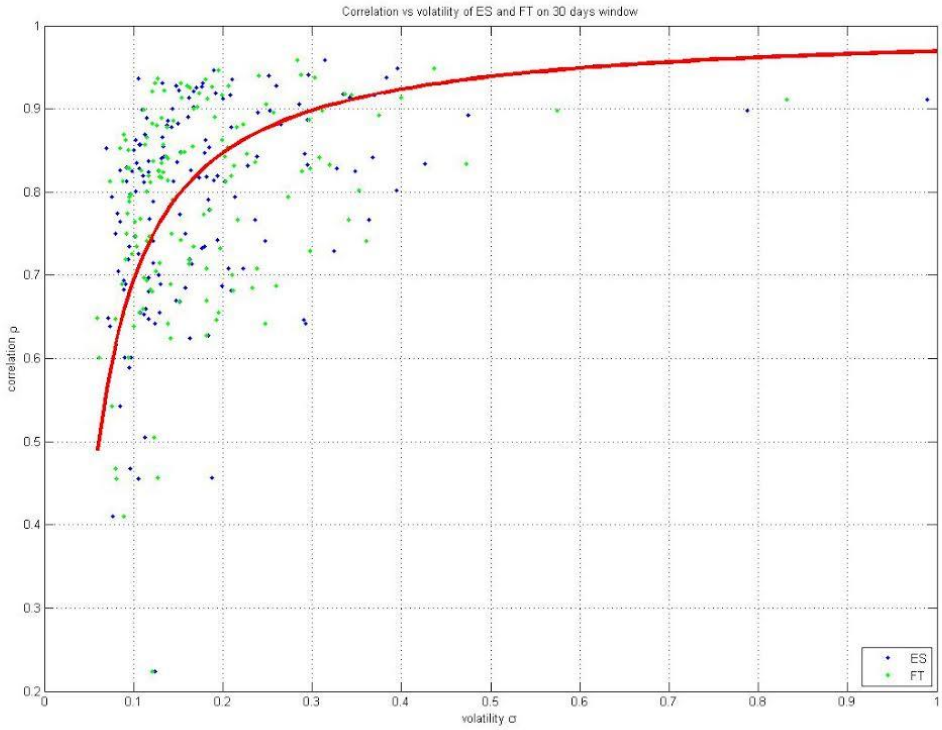
It is also known that the correlation is not stable, as the price data series are not perfectly stationary. Same holds for volatilities - they are believed to be functions of time. The exact nature of the relationship between the two measurable functions of correlation and volatilities is believed to be unknown.

In this experiment, we wanted to accurately measure the statistical relationship between the correlation and market volatilities as a function of time in the most accurate fashion.

For this purpose we have used the historical data for both markets from 12/1998 to 4/2016 at daily resolution, which was synchronized exactly where both markets have good liquidity, namely at 11:30AM EST. We have measured the correlation and each market's volatility, annualized over a certain window: 15 and 30 days in what follows. These intervals of time (15 and 30 days) were chosen in non-overlapping fashion, so that each 15- or 30-day interval produces three numbers: correlation, volatility of the first market and volatility of the second market. We would consider them as observations of these quantities over time.

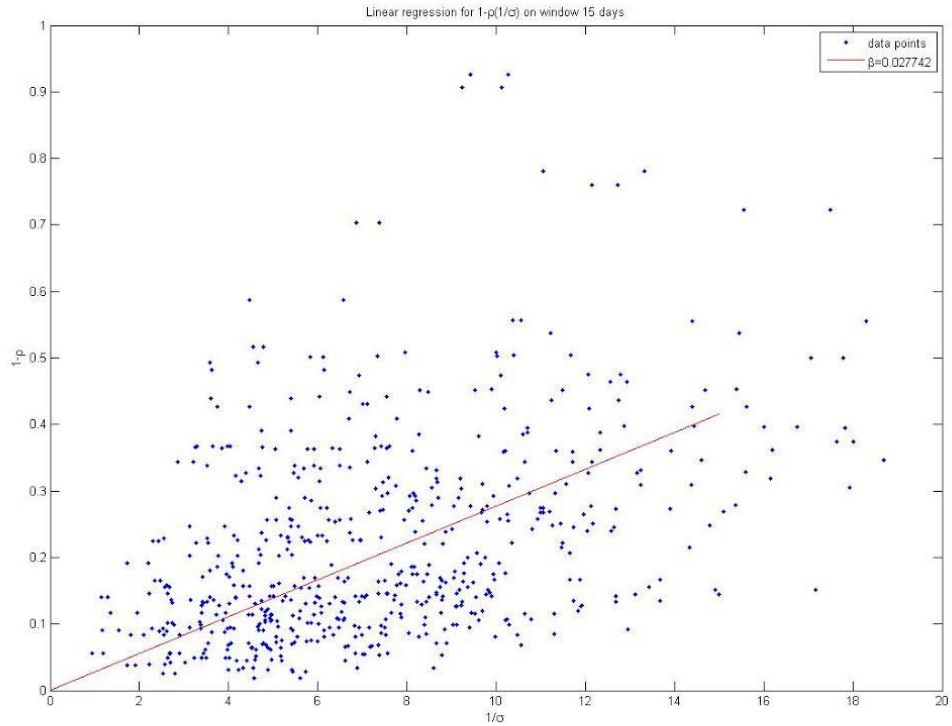
Some knowledge of their relationship can be gained from their scatter plot, and by proposing a simple regression which may be a good fit to that scatter plot. We have chosen two intervals of 15- and 30-days to see the dependence of the results on that window size which, as one may see, is small.

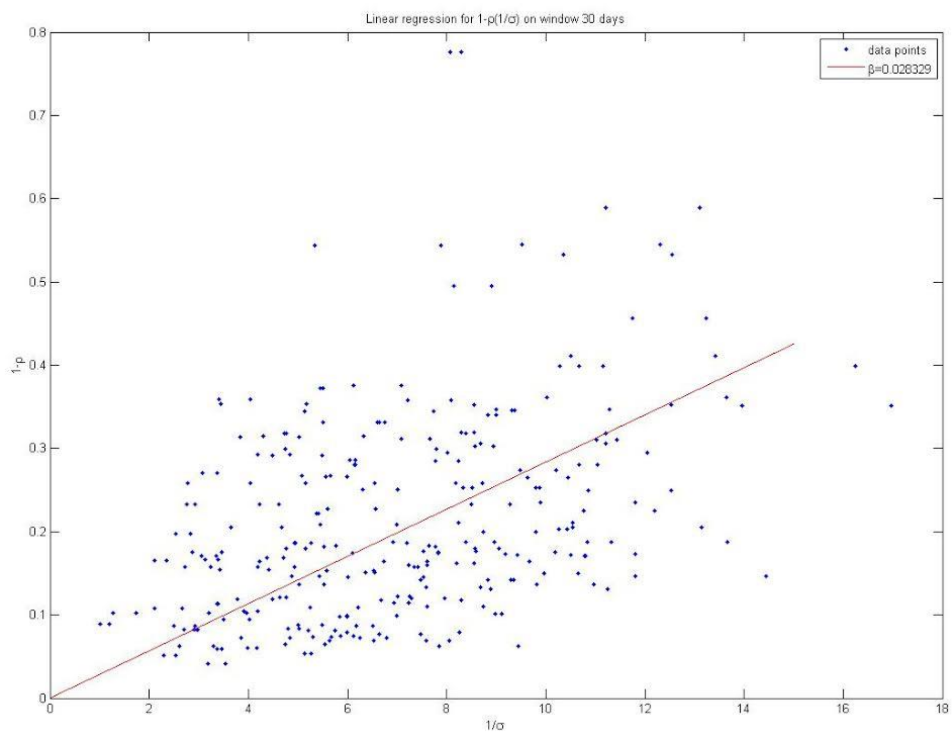




As one can see from the above scatter plots, a very clear relationship between the correlation ρ and market volatility σ is observed. Namely, the more volatile the markets are, the more correlated they become. For example, for points with volatilities above 40% one can see correlation coefficients above +80%.

Additionally, we can try to guess a simple algebraic relationship which may fit this statistical relationship: correlation coefficient ρ is inversely proportional to volatility σ. For that we have regressed the 1- ρ onto the 1/ σ for the same above two cases. The corresponding graphs are shown below.





Using the regression slopes we have inferred from these charts, we have plotted the thick fitted lines in the first two graphs above.

The results of this experiment illustrate very clearly the nature of the relationship between correlation and volatility: **markets become more correlated in more volatile environments**. Even though the reverse also seems to be true (i.e., markets are less correlated in low volatility environments), most correlations are still above +50% even during periods of very low volatilities, with approximately only 3.4% of all observations having correlations below +50%.